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SLUGULATION DURING EXPLOSION DUE TO TAYLOR INSTABILITY

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ABSTRACT

The time development of irregularities due to the "Taylor Instability" is calculated for the two-dimensional case. The rate of slugulation is calculated for two special types of irregularities. The application of the results to the efficiency of explosion of nuclear bomb is pointed out.

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SLUGULATION DURING EXPLOSION DUE TO TAYLOR INSTABILITY

Taylor has called attention to a hydrodynamical theorem which states that the acceleration of a more dense material into a less dense material causes pre-existent irregularities at the interface between the two materials to grow[1)] until mixing takes place. Mixing due to the so-called "Taylor instability" plays a role in the implosion of the bomb, especially in connection with certain proposed types of initiation, but more important, its effect on the efficiency of the explosion must be evaluated. Thus, when a bomb explodes the active material expands and a shock wave is sent into the tamper; this shock wave has a much higher density than the untouched tamper in front of it. The conditions for a "Taylor instability" are, therefore, present and it is necessary to know the time development of irregularities in the shock which may have arisen, say, from an asymmetric implosion. After a sufficiently long time, slugs of tamper material will be formed and mix in with the active material with a consequent loss of efficiency. A knowledge of the rate of slugulation and the maximum amount of tamper material which can be transformed into slugs under different initial conditions will permit more realistic calculations to be made of the bomb efficiency. Since the average density in the shock wave is at least ten times the density of the unshocked tamper, this is true in the early stages even with radiation- it is possible to regard the shock wave as possessing infinitessimal thickness. Such an approximation is equivalent to the assumption that $\gamma = 1$ (γ is the ratio of specific heats at constant pressure and volume and is usually about 1.4) and should apply when the wave length of the irregularity is large compared to the thickness of the shock wave[2)]. With t

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1). If ρ_1 and ρ_2 are the densities of the two materials ( ), k is the wave number of the irregularity, and a is the relative acceleration, then the "characteristic time of growth" is $\sqrt{\left(\frac{\rho_2+\rho_1}{\rho_2-\rho_1}\right)\frac{1}{ka}}$

2). The mixing phenomena associated with the finite thickness of the shock wave are investigated elsewhere by Bowers and Weisskopf.

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With the assumption, $\gamma = 1$, and the further restriction that the irregularity is two-dimensional, it is simple to write down the equation of continuity and the equations of motion. Suppose m is the mass per unit Lagrangian arc length S_o (i.e., S_o is the arc length measured from some reference point at the initial time),

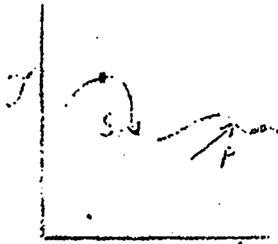

FIG 1

p is the pressure defined as force per unit Eulerian arc length, σ_o is the normal density per unit area and $x(S_o, t)$, $y(S_o, t)$ are the Eulerian coordinates of a mass point S_o at time t; then we get (cf. Fig. 1):

$$\frac{\partial m}{\partial t} = \sigma_o \left[\frac{\partial y}{\partial t} \frac{\partial x}{\partial S_o} - \frac{\partial x}{\partial t} \frac{\partial y}{\partial S_o} \right] \tag{1}$$

$$\frac{\partial}{\partial t} \left(m \frac{\partial y}{\partial t} \right) = \rho \frac{\partial x}{\partial S_o} \tag{2}$$

$$\frac{\partial}{\partial t} \left(m \frac{\partial x}{\partial t} \right) = -\rho \frac{\partial y}{\partial S_o} \tag{3}$$

Eq. (1) is obvious and relates the increase in m per unit time to the area swept up per unit time per unit S_o. Eqs. (2) and (3) are the equations of motion and relate the increase in the components of momenta to the appropriate components of pressure- where the pressure is presumed to act normally to the shock-wave contour at all points and for all time.

For our purposes, it is necessary to assume that the pressure at the back of the shock wave increases exponentially with time. i.e. $p = p_o e^{2\alpha t}$ where p_o and α are constants. Introducing a new variable $\tau = e^{\alpha t}$, denoting differentiation with respect to τ by a dot and with respect to S_o by a prime and choosing the unit of pressure so that $p_o/\alpha^2 = 2$, equations (1) - (3) become:

$$\dot{m} = \sigma_o \left(\dot{y} x' - \dot{x} y' \right) \tag{4}$$

$$(m \tau \dot{y}) = 2 \tau x' \tag{5}$$

$$(m \tau \dot{x}) = -2 \tau y' \tag{6}$$

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When no irregularity is present, equations (2) to (3) should lead to the well-known result for a plane shock wave with $\gamma = 1$, namely that the pressure at the front of the shock wave, i.e., $\sigma_0\, y\, \partial^2 y/\partial t^2$ is one half the pressure at the back. That this is so follows immediately from (4) - (6): $x = S_0 \equiv$ const, so that (4) yields $m = \sigma_0\, y$ and (5) $(y\dot{y}) = \tau/\sigma_0$; we have made use of the fact that m and y are zero when $\tau = 0$. Integrating the second equation again leads to $y = \tau/\sqrt{\sigma_0}$. Consequently $\sigma_0\, y\, \partial^2 y/\partial t^2 =$

$\rho_0\, e^{2\alpha t}/2$ and the front pressure is one-half the back pressure.

Eqs. (4) to (6) cannot be solved analytically for an arbitrary initial irregularity $\chi(S_0, 0) = \chi_0(S_0)$, $y(S_0, 0) = y_0(S_0)$. ($\tau = 0$ corresponds to $t = -\infty$, which is taken as the initial time.) However, an expansion in a power series in τ turns out to be quite convenient. We write:

$$\left.\begin{aligned} x(S_0,\tau) &= x_0 + x_0'\,\psi - y_0'\,\phi \\ y(S_0,\tau) &= y_0 + x_0'\,\phi + y_0'\,\psi \end{aligned}\right\} \qquad (7)$$

where ϕ and ψ are functions of S_0 and τ determined by Eqs. (4) to (6). That the representation (7) is permissible can easily be checked by substitution into (5) and (6); we have

$$\dot{y} = x_0'\,\dot{\phi} + y_0'\,\dot{\psi}$$

$$x' = x_0' + (x_0''\psi + x_0'\psi') - (y_0''\phi + y_0'\phi')$$

Inserting into (5), we get:

$$\left\{m\,\tau\,(x_0'\dot{\phi} + y_0'\dot{\psi})\right\}^{\cdot} = 2\,\tau\left[x_0' + (x_0''\psi + x_0'\psi') - (y_0''\phi + y_0'\phi')\right] \qquad (8)$$

If we define the radius of curvature at any point S_0 as $R_0(S_0) = x_0'/y_0''$, it follows from the identity $x'^2 + y'^2 = 1$ that R_0 is also $(-y_0'/x_0'')$. Substituting for x_0'', y_0'' into (8) and rearranging terms, we get:

$$x_0'\left[m\,\tau\,\dot{\phi}\right]^{\cdot} + y_0'\left[m\,\tau\,\dot{\psi}\right]^{\cdot} = 2\,\tau\, x_0'\left[1 + \psi' - \phi/R_0\right] + 2\,\tau\, y_0'\left[\phi' + \psi/R_0\right] \qquad (9)$$

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Equating the coefficients of x_0' and y_0' in (9), we find for ϕ, ψ the coupled differential equations:

$$(m\dot{\tau}\dot{\phi}) = 2\tau(1 + \psi' - \phi/Ro) \qquad (10)$$

$$(m\dot{\tau}\dot{\psi}) = -2\tau(\phi' + \psi/Ro) \qquad (11)$$

An identical procedure applied to Eq. (6) leads immediately to the same Eqs. (10) and (11)
Eqs. (10) and (11) are the equations which are to be solved together with (4) which can be rewritten as:

$$m = \sigma_0\left[\phi - \frac{1}{2R_o}(\phi^2 + \psi^2) + \int_0^\tau (\dot{\phi}\psi' - \dot{\psi}\phi')\, d\tau\right] \qquad (12)$$

We now expand ϕ and ψ as power series in τ, i.e.

$$\phi(S_0, \tau) = \sum_{n=1}^{\infty} \phi_n(S_0)\, \tau^n \qquad (13)$$

$$\psi(S_0, \tau) = \sum_{n=1}^{\infty} \psi_n(S_0)\, \tau^n \qquad (14)$$

The series start with $n = 1$ since $m = 0$ when $\tau = 0$.

Evaluation of the ϕ_n's and ψ_n's proceeds in the customary manner and we find that the first two ψ_n's vanish. Thus we may write up to third order:

$$\left.\begin{aligned} x &= x_0 - y_0'\phi \\ y &= y_0 + x_0'\phi \end{aligned}\right\} \qquad (15)$$

Eq. (15) asserts that the motion of every point of the irregular surface is normal to itself. The fact that ψ only enters in third order can be used to obtain very simply the higher coefficients. We have calculated the first six sets of coefficients with the following results:

$$\phi_1 = 1 \qquad \psi_1 = 0$$

$$\phi_2 = \frac{-.0556}{R_o^2} \qquad \psi_2 = 0$$

$$\phi_3 = -\frac{.0224}{R_o^2} \qquad \psi_3 = 9.26 \cdot 10^{-3}\left(\frac{1}{R_o}\right)'$$

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$$\phi_4 = 2.88\cdot10^{-4}\left(\frac{1}{R_o}\right)'' - \frac{9.16\cdot10^{-3}}{R_o^3} \qquad \psi_4 = \frac{7.41\cdot10^{-3}}{R_o}\left(\frac{1}{R_o}\right)' \tag{16}$$

$$\phi_5 = \frac{3.81\cdot10^{-4}}{R_o}\left(\frac{1}{R_o}\right)'' - \frac{3.53\cdot10^{-3}}{R_o^4} \qquad \psi_5 = \frac{4.44\cdot10^{-3}}{R_o^2}\left(\frac{1}{R_o}\right)' - 1.85\cdot10^{-5}\left(\frac{1}{R_o}\right)$$

$$+ 1.13\cdot10^{-4}\left[\left(\frac{1}{R_o}\right)'\right]^2$$

$$\phi_6 = \frac{3.39\cdot10^{-4}}{R_o^2}\left(\frac{1}{R_o}\right)'' - \frac{1.30\cdot10^{-3}}{R_o^5} \qquad \psi_6 = \frac{2.30\cdot10^{-3}}{R_o^3}\left(\frac{1}{R_o}\right)' - \frac{2.58\cdot10^{-5}}{R_o}\left(\frac{1}{R_o}\right)'''$$

$$+ \frac{1.99\cdot10^{-4}}{R_o}\left[\left(\frac{1}{R_o}\right)'\right]^2 - 3.15\cdot10^{-7}\left(\frac{1}{R_o}\right)^{(iv)} - 4.09\cdot10^{-5}\left(\frac{1}{R_o}\right)'\left(\frac{1}{R_o}\right)''$$

We now wish to apply our results to the exploding gadget and study the slug development of the shock wave in the tamper due to irregularities in the original interface. It is clear-and can immediately be derived from Eqs. (10) to (12) - that one-half of the tamper material swept over by the shock wave will be concentrated in slugs when the shock wave has traveled a distance large compared to the amplitude of the original irregularities. The asymptotic value 1/2 arises from the assumptions that (a) the shock wave and the initial irregularities on the interface are all plane; (b) the shock wave is infinitely compressed, and (c) the pressure behind the shock wave is spatially constant. If assumption (a) is lifted and the three-dimensional case is considered, the value 1/2 becomes 2/3.

While it is certainly worthwhile to know the maximum amount of slugulation, it is also of interest to investigate how soon the asymptotic value is reached for different types of irregularities. Since the exact nature of the irregularity which may be present on the interface at the start of the explosion is not known, we have studied two simple irregularities, namely:

$$y_o = \cos x_o \tag{17a}$$

$$y_o = .1 \cos x_o \tag{17b}$$

The curve (17a) or (17b) is assumed to be the configuration of the infinitely compressed shock wave at $\tau = 0$ and we have computed y as a function of x for increasing τ using (16). The results for (17a) are shown in Fig. 1; similar results are obtained for (17b).

It is seen from Fig. 1 that as time goes on, more and more of the tamper material hits the line $x = 3\pi / 2$ to form slugs. It is found that 30 % of the material is slugulated after a displacement of 2.5 times the initial amplitude; this is to be compared with the asymptotic value of 50 %. In the case (17b), where the wave length is the same and the initial amplitude is 1/10 as great, 30 % of the tamper material is slugulated after a displacement of 100 times the initial amplitude.

The above results demonstrate that the maximum amount of slugulation is reached rather late and relatively later, the smaller the initial amplitude. The effect of the slugulation process on the efficiency of the explosion is certainly unfavorable - since less mass is moved by the same pressure - but a quantitative estimate is only possible if the distribution of slugs behind the shock wave and their radiation heating are known. The latter points are being investigated by Bowers and Weisskopf.

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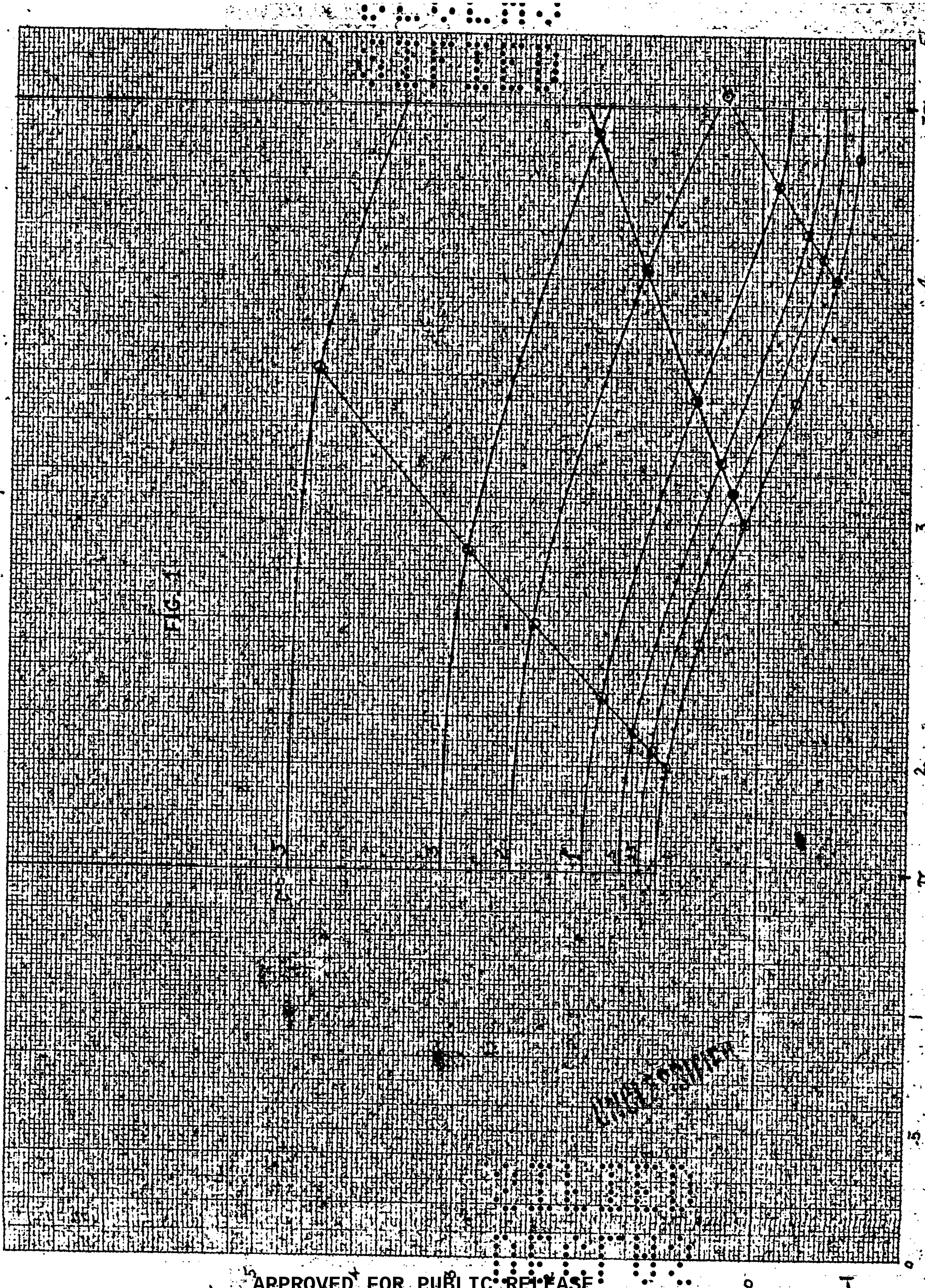

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